ROSSVIEW AP Calculus summer assignment

1. If n is of the form a^2 and a is an integer, which of the following MUST be an odd integer?

(A) $n + 1$

(B) $n + 2$

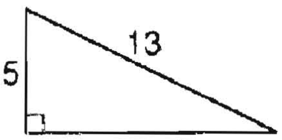
(C) $2n + 1$

(D) $3n$

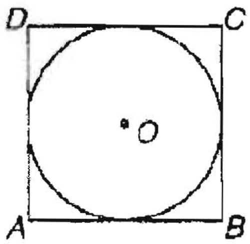
(E) $3n + 1$

Express 2b in terms of a

2. If $\dfrac{a}{b+1} = 2$, what does $2b$ equal?



3. What is the value of x in the diagram above?



4. Circle O above is inscribed inside square $ABCD$. If the area of the circle is 9π, what is the perimeter of the square?

"Some marbles in the bag are green."

If the statement above is true, which of the following must also be true?

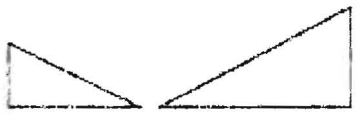
(A) If a marble is in the bag, it is green.

(B) If a marble is green, it is in the bag.

(C) Some marbles in the bag are red.

(D) Some marbles in the bag are not red.

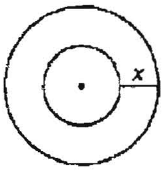
5. (E) There are more green marbles in the bag than any other color.



6. The triangles above are similar. If the areas of the two triangles are 16 and 36 respectively, what is the ratio of their perimeters?

7. If $f(x) = \frac{1}{2}x - c$ and $f(8) = 6$, what is the value of c?

8. If $\left(3^{\frac{n}{2}}\right)^{-8} = 3$, what is the value of n?



9. If the circumference of the larger circle above is 10π, and the circumference of the smaller circle is 6π, what is the value of x?

10. If a circle with radius 5 has its center at the point $(-1, 3)$, which of the following points is on the circle?

(A) $(-6, -2)$

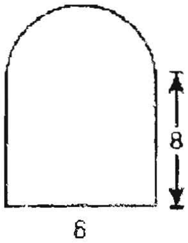
(B) $(-1, -2)$

(C) $(-4, 8)$

(D) $(6, 3)$

(E) $(4, 8)$

Give an exact answer in terms of π



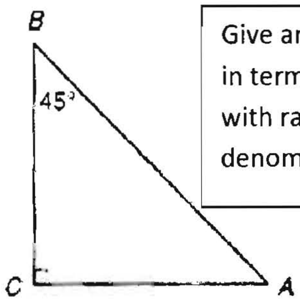
11. The shape above was made by connecting a semicircle to a rectangle. What is the perimeter of the shape?

12. If $x + y = 1.2$, then $x^2 + 2xy + y^2 =$

13 If $a \Phi b = \frac{a+b}{a-b}$, then $\frac{1}{2} \Phi 1 = ?$

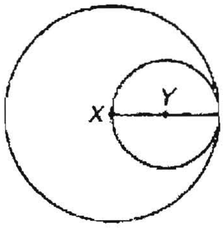
Must show work on paper: do fractions without calculator

14 What is the value of a so that the line that passes through the points $(a, 7)$ and $(5, 4)$ is perpendicular to the line $y = 2x + 7$?



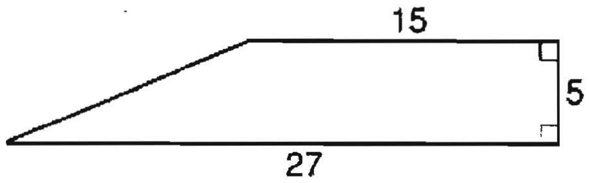
Give an exact answer in terms of radicals with rational denominator

15 The ratio $AB : BC$ equals

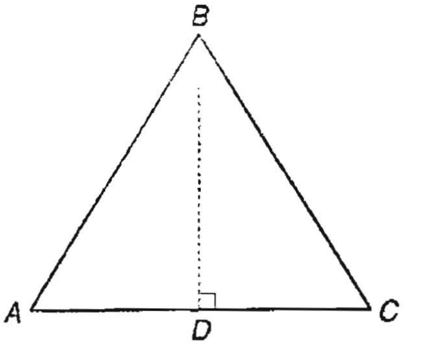


16 The diameter of ⊙X above is 40 inches. What is the ratio of the area of ⊙Y to the area of ⊙X?

17 Ramon can paint a fence in two hours. Jenny can paint the same fence 15 minutes faster than Ramon and Dennis can paint it in half Ramon's time. About how many minutes will it take to paint the fence if all three people work together?



18. What is the perimeter of the trapezoid shown above?



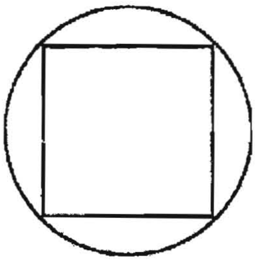
19 The slope of $\overline{AB}$ is $\frac{2}{5}$, the slope of $\overline{BC}$ is $-\frac{2}{5}$, and the length of $\overline{AC}$ is 20. What is the length of $\overline{BD}$?

Give an exact answer in terms of radicals with rational denominator

20 Which of the following is the same as $\frac{1}{\sqrt{4x}} + \frac{\sqrt{x}}{2}$?

21. Mike can cut a 40 foot by 20 foot yard with a push mower in 25 minutes. At this rate, how many <u>more</u> minutes would it take him to cut a 40 foot by 30 foot yard?

22. What is the ratio of the area of the square to the area of the circle if the length of a side of the square is 10 units?

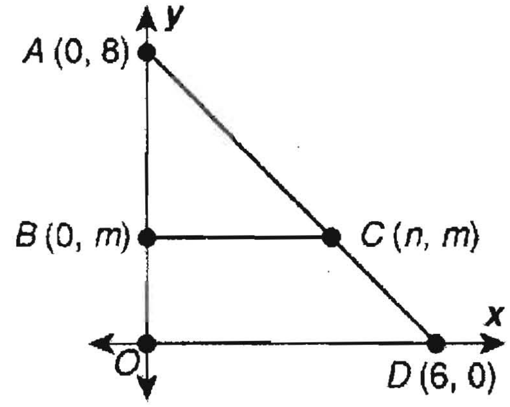


23 $\log\left(\frac{1}{10^a}\right)$ =? express in terms of a

24 $4x^3 + 12x^2 + 9x + 27 = 0$ find all three roots including imaginary

25 A sphere is inscribed inside a cube. What is that probability that a point that is inside the cube is also inside the sphere?

26 Solve for x: $\sqrt{ax+1} - \sqrt{ax-1} = \sqrt{ax}$. express x in terms of a



27

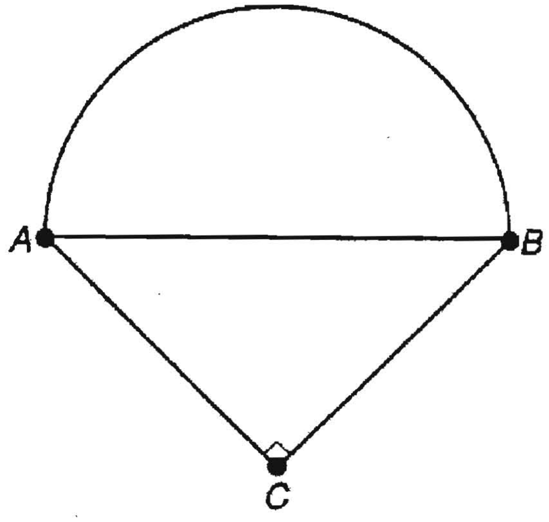
Find the value of m so that the area of the triangle ABC equals the area of trapezoid OBCD

Points A and C lie on a straight road and point B lies directly above the road.
The angle of elevation from point A to point B is 35° and the angle of depression from point B to point C is 35°. If the distance from A to C is 20 miles, approximately how many
28 miles above the road is point B?

Express answer rounded to hundredths of a mile

If $\dfrac{2x-3}{3x^2+16x+5} + A =$

29 $\dfrac{3x^2+3x+18}{3x^3+13x^2-11x-5}$, then $A =$



The figure shows a semicircle on top of an isosceles right triangle. If the length of $\overset{\frown}{AB}$ is
30 16π, what is the approximate length of $\overline{BC}$?

If $g(x) = \dfrac{5x-3}{2x^2-11-6}$, what is the sum of all the real numbers that are not in the
31 domain of $g(x)$?

The zeros of $m(x) = \dfrac{x^2 + 3x + 2}{x^2 - 3x + 2}$ are the first two terms of a sequence. Each term in the sequence is found by adding the two terms before it. If each term is smaller than the one before it, what is the fifth term of the
32 sequence?

What is an equation of the circle that has its center at the origin and is tangent to the line
33 $y = -3x + 7$?

If $h(x) = (f \circ g)(x)$ and $h^{-1}(x)$ is the inverse of $h(x)$, then which of the following must be equal to x?

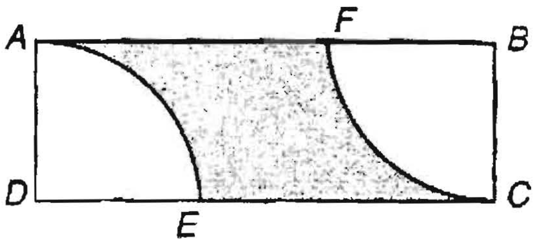
(A) $(h^{-1} \circ f \circ g)(x)$

(B) $(h^{-1} \circ g \circ f)(x)$

(C) $(f^{-1} \circ g^{-1} \circ h^{-1})(x)$

34 (D) $(g^{-1} \circ f^{-1} \circ h^{-1})(x)$

Figure $ABCD$ is a rectangle whose length is twice its width. $\overset{\frown}{FC}$ and $\overset{\frown}{AE}$ are arcs of circles centered at B and D respectively. If the length of $\overline{AD}$ is x, then the area of the
35 shade region is



36

37. If $3x - 2 = y$, What is $y^2 + 6$?

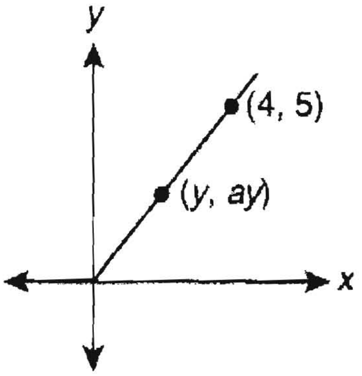
The length of the base of a certain television screen is 50 inches and it makes a $35°$ angle with the diagonal of the screen. Approximately how long, in inches, is the
38 diagonal?

39 For what values of x is $|2x + 1| + 3 < 8$?

If $4m^2 + 9n^2 = 1$ and $(2m - 3n)^2 = 13$,
40 what is the value of mn?

40 Find $f(3)$ if $f(x) = \begin{cases} 2 - x, & \text{if } x > 3 \\ x + 2, & \text{if } x \leq 3 \end{cases}$

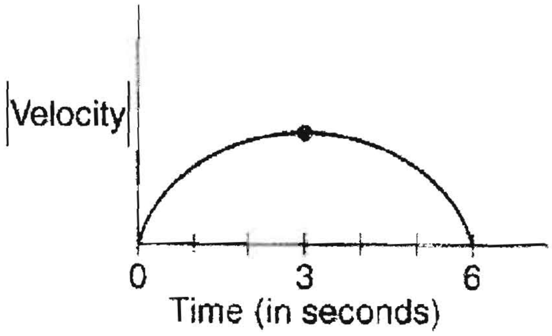
41 A tree fell over and is now leaning against the top of Mrs. Collini's house. The height of the house is 20 meters and the base of the tree is 35 meters from the base of the house. What angle does the fallen tree make with the ground, rounded to the nearest degree?

42 

find a

Remember that the absolute value (magnitude) of velocity is speed. Give the intervals where the speed is increasing, decreasing, and the point where the speed is maximum.

43 A car is traveling on a straight road. Its |velocity| versus time is shown on the graph.



44. If $5x - 6 = 3(y - 2)$, then $5\left(\frac{x}{y}\right)$ is

45 *if* $\sin\left(x - \frac{\pi}{2}\right) = a$ *find* $\cos(x)$ in terms of a.

45 What is the range, in interval notation, of the piecewise function?

$$g(x) = \begin{cases} -3x + 5, & \text{if } -4 \leq x \leq 0 \\ 3x + 6, & \text{if } 0 < x \leq 4 \end{cases}$$

46. Express in terms of a . round to 2 decimal places

If $\sin(ax) = 0.3\cos(ax)$, x is in radians, $-\frac{\pi}{2} < x < \frac{\pi}{2}$, and $a > 1$, then $x =$

47 Line A has the equation $2x + 3y = 7$. If line B is perpendicular to line A at $x = 2$, where does line B intersect the x-axis?

48 The function $f(x)$ has the following values: $f(1) = 2$, $f(2) = 5$, $f(5) = 8$, and $f(8) = 10$. If $f^{-1}(x)$ is the inverse of $f(x)$, then $f^{-1}(5) =$

If a and b are greater than 0, then

49 $\sin(\arctan \frac{a}{b}) =$ express in terms of a and b

If $f(x) = -a\sin(bx + c) + d$ and $a, b, c, d > 0$, what is the range of $f(x)$ in interval notation?

50. express in terms of a, b, c, and or d

x	y
−3	0
−2	1
−1	1.5
0	1
1	3
2	4

51. Let $f(x)$ be the equation of the line-of-best-fit used to approximate the data given in the chart. What is the approximate value of $f(5)$?

52, Let f(x) be the exponential regression equation for the data table below.

Use a graphing calculator to find the regression equation." Stat button"edit "enter". Put numbers in L1 and L2. " Stat button" over to "calc" down to expreg "enter" NOW before you hit enter again. "VARS button(by clear)" over to "y vars" over to "function" Y1"enter"

Screen should look like

ExpRegY1

Hit enter.

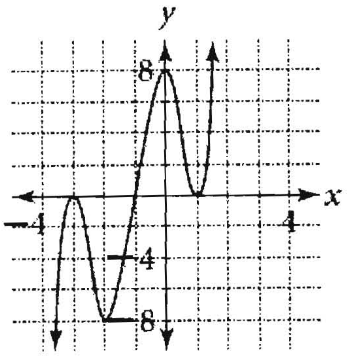
This automatically puts the equation into "y=" . Now go to TBLSET ("2nd window" and change the ΔTbl to 0.1. Now hit "2nd graph"(TABLE) to see a table of values. Use this table to answer the questions below.

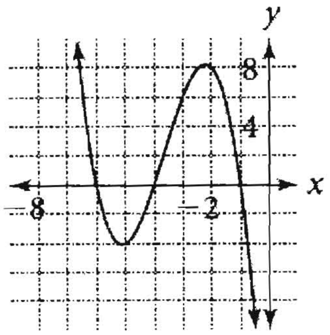
Years past 1980	# of computers in USA
0	23000
5	103500
10	527850
15	2216970
20	12415032
25	48418625

What is the meaning of f(12)? Explain in a short sentence. Give the numerical value for f(12).

What is the meaning of $f^{-1}(316742)$? Explain in a short sentence. Give the numerical value for $f^{-1}(316742)$

53

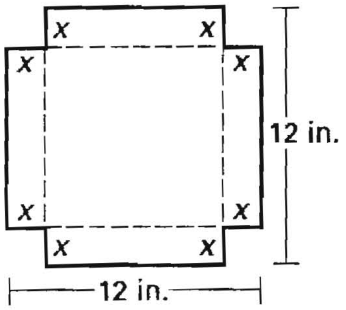




For each of the graphs to the left, express a polynomial in order of degreasing exponents that closely matches the graph in terms of intercepts, maxima, and minima. List local maxima, minim, and intervals where the function is increasing and decreasing.

54

The corners of a square piece of cardboard are cut out and the sides folded up to form an open box as shown in the figure below.



Let f(x) be a function that gives the volume of the box shown and described to the left. Give the range of f(x)

55 Solve the equation $be^{ax} \cdot e^{c} = 1$ for x.

56. Find the reference angle for x=$\frac{53\pi}{3}$. Find the x and y coordinates of the point on the unit circle associated with x. an angle on the interval (0,2π] that is coterminal with x. Evaluate all six trig functions for x. Express answers exactly using radicals.

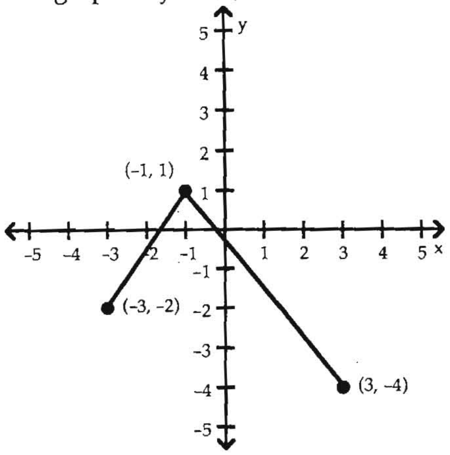
57 On the interval $[-\frac{7\pi}{2}, -\frac{3\pi}{2})$,find all sub-intervals for which the function f(x) = sin(4x) is decreasing.

Solve the problem.

The graph of $y = x^2$ is shifted to the right by 3 units. Write the resulting equation.

Using transformations, sketch the graph of the function.

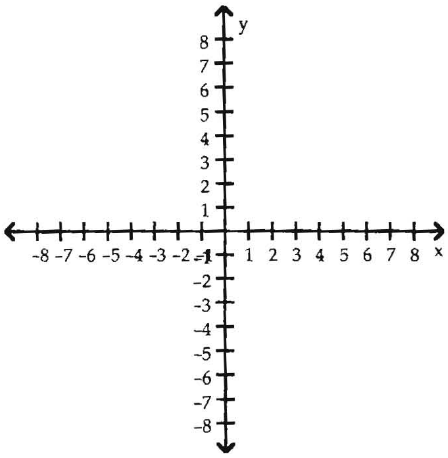
58 The graph of $y = f(x)$ is as shown. Sketch the graph of $y = f(x + 2) - 1$



Graph the function.

) Graph the function whose graph is that of $y = x^3 - x^2 - 6x$ but is reflected about the y-axis.

59



Find the domain of the composite function $f \circ g$. Give the composite function for 60-62

60) $f(x) = 6x + 54; \quad g(x) = x + 1$

61) $f(x) = \frac{8}{x+8}; \quad g(x) = x + 5$

62) $f(x) = \sqrt{x-1}; \quad g(x) = \frac{1}{x-6}$

Solve the problem.

63) An oil well off the Gulf Coast is leaking, with the leak spreading oil over the surface of the gulf as a circle. At any time t, in minutes, after the beginning of the leak, the radius of the oil slick on the surface is $r(t) = 6t$ ft. Find the area A of the oil slick as a function of time.

64) Let $f(x) = \sqrt{2-x}$ and $g(x) = |2x - 1|$. Find the domain of $(f \circ g)(x)$. Express answer in interval notation.

65) Let $g(x) = \frac{x-1}{x+2}$ and $h(x) = 4x - 3$. Find $(h \circ g)(x)$. Express answer as a single fraction in reduced form.

Find functions f and g so that the composition of f and g is H.

66) $H(x) = \sqrt[3]{x+1}$

Solve the problem.

67) A wire of length 6x is bent into the shape of a square. Express the area of the square as a function of x.

68) Two boats leave a dock at the same time. One boat is headed directly east at a constant speed of 35 knots (nautical miles per hour), and the other is headed directly south at a constant speed of 22 knots. Express the distance *d* between the boats as a function of the time *t*.

69) An open box with a square base is required to have a volume of 27 cubic feet. Express the amount *A* of material used to make such a box as a function of the length *x* of a side of the base.

70) Let P(x, y) be a point on the graph of $y^2 = 4x + 4$. Express the distance, d, of the point P from the origin. Express your answer in a simplified form.

In the problem, t is a real number and P=(x,y) is the point on the unit circle that corresponds to t. Find the exact value of the given trigonometric function.

71) $\left(\frac{5}{6}, \frac{\sqrt{11}}{6}\right)$; find sin t

72) $\left(\frac{5}{6}, \frac{\sqrt{11}}{6}\right)$; find tan t

73) $\left(\frac{\sqrt{55}}{8}, \frac{3}{8}\right)$; find sec t

74) $\left(-\frac{\sqrt{33}}{7}, \frac{4}{7}\right)$; find cos t

75) $\left(-\frac{\sqrt{65}}{9}, \frac{4}{9}\right)$; find cot t

A point on the terminal side of angle θ is given. Find the exact value of the given trigonometric function.

76) (12, 16); Find sin θ.

77) (6, 8); Find cos θ.

78) (-15, 36); Find sin θ.

79) (21, 28); Find csc θ.

Find the value of the expression.

80) $\sin^{-1} \frac{\sqrt{2}}{2}$

81) $\cos^{-1} \frac{\sqrt{2}}{2}$

82) $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

83) $\tan^{-1} -1$

84) $\sin^{-1} -0.5$

85) $\tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$

Use a calculator to find the value of the expression in radian measure rounded to 2 decimal places.

86) $\sin^{-1}(0.4)$

87) $\cos^{-1}\left(\frac{1}{6}\right)$

88) $\tan^{-1}(1.5)$

89) $\sin^{-1}\left(\frac{\sqrt{5}}{3}\right)$

Solve the problem.

90) A building 150 feet tall casts a 80 foot long shadow. If a person looks down from the top of the building, what is the measure of the angle between the end of the shadow and the vertical side of the building (to the nearest degree)? (Assume the person's eyes are level with the top of the building.)

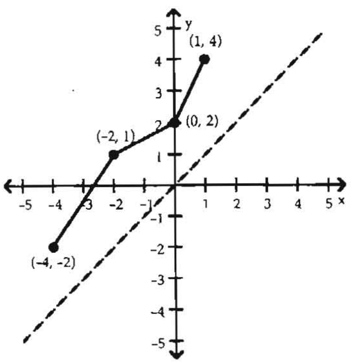
91) Two surveyors 180 meters apart on the same side of a river measure their respective angles to a point on the other side of the river and obtain 54° and 68°. How far from the point (line-of-sight distance) is each surveyor? Round your answer to the nearest 0.1 meter.

92) A hill slopes at an angle of 15° with the horizontal. From the base of the hill, the angle of elevation of a 600 ft tower at the top of the hill is 40°. How much rope would be required to reach from the top of the tower to the bottom of the hill? Round answer to the nearest foot.

93) A room in the shape of a triangle has sides of length 7 yd, 10 yd, and 15 yd. If carpeting costs $18.50 a square yard and padding costs $4.25 a square yard, how much to the nearest dollar will it cost to carpet the room, assuming that there is no waste?

Use the graph of the given one-to-one function to sketch the graph of the inverse function. For convenience, the graph of y = x is also given.

94)



MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

The function f is one-to-one. Find its inverse.

95) Determine the equation for the inverse function of $y = (x+2)^3 - 8$.

A) $y = \sqrt[3]{x+8} - 2$ B) $y = \sqrt[3]{x-2} + 8$ C) $y = \sqrt[3]{x+10}$ D) $y = \sqrt[3]{x+6}$

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Solve the problem.

96) A rumor is spread at an elementary school with 1200 students according to the model $N = 1200(1 - e^{-0.16d})$ where N is the number of students who have heard the rumor and d is the number of days that have elapsed since the rumor began. How many students will have heard the rumor after 5 days?

Write the logarithmic expression as a sum and difference of logarithms.

97) $\log_a \dfrac{x^4 \sqrt[3]{x+5}}{(x-2)^2}$

Use the Change-of-Base Formula and a calculator to evaluate the logarithm. Round your answer to two decimal places.

98) Evaluate $\log_{(2/3)} 19$.

Use the Change-of-Base Formula and a calculator to evaluate the logarithm. Round your answer to three decimal places.

99) $\log_5 50.48$

Solve the equation.

100) $\log_2 x = 5$

101) $\log 4x = \log 2 + \log (x - 1)$

102) $\log (3 + x) - \log (x - 4) = \log 2$

Solve the given logarithmic equation.

103) $\log_2(3x - 2) - \log_2(x - 5) = 4$

104) $2 + \log_3(2x + 5) - \log_3 x = 4$

Solve the equation. If necessary, round your answer to two decimal places.

105) $3^x = 27$

106) $4^{(1 + 2x)} = 64$

107) $(\frac{1}{4})^x = 14$

108) $4^{(2x - 1)} = 16$

Solve the given exponential equation.

109) $3 \cdot 5^{2t - 1} = 75$

Find the present value.

110) How much money need to be invested now to get $2000 after 4 years at 8% compounded quarterly? Express your answer to the nearest dollar.

Solve the problem.

111) A culture of bacteria obeys the law of uninhibited growth. If 140,000 bacteria are present initially and there are 609,000 after 6 hours, how long will it take for the population to reach one million?

112) The half-life of silicon-32 is 710 years. If 40 grams is present now, how much will be present in 200 years? (Round your answer to three decimal places.)

113) A fossilized leaf contains 14% of its normal amount of carbon 14. How old is the fossil (to the nearest year)? Use 5600 years as the half-life of carbon 14.

114) The logistic growth model $P(t) = \frac{1240}{1 + 40.33e^{-0.325t}}$ represents the population of a bacterium in a culture tube after t hours. What was the initial amount of bacteria in the population?

115) The logistic growth model $P(t) = \frac{180}{1 + 44e^{-0.188t}}$ represents the population of a species introduced into a new territory after t years. When will the population be 80?

Use the fact that the trigonometric functions are periodic to find the exact value of the expression.

116) $\cos \frac{10\pi}{3}$

Give the amplitude or period as requested.

117) Amplitude of $y = 4 \sin x$

118) Amplitude of $y = -3 \sin 5x$

119) Period of $y = \sin 5x$

120) Period of $y = -5 \cos \frac{1}{2}x$

121) Period of $y = \frac{5}{4} \sin \left(-\frac{4\pi}{7}x\right)$

Use the fact that the trigonometric functions are periodic to find the exact value of the expression.

122) $\sin 765°$

123) $\sin \frac{10\pi}{3}$

124) $\tan 390°$

125) $\tan \frac{13\pi}{4}$

126) $\csc 660°$

127) $\sec \frac{13\pi}{4}$

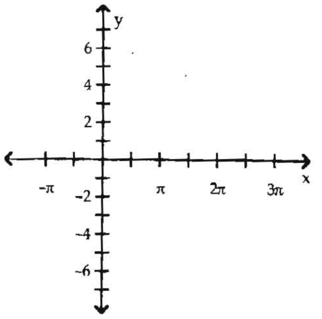
128) $\cot 570°$

Find the amplitude, period, and phase shift of the sinusoidal funtion.

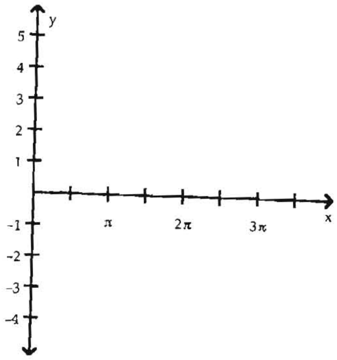
129) $y = -\frac{3}{4} \sin\left(\frac{1}{4}x + \frac{\pi}{2}\right)$

Graph the function. Show at least one period.

130) $y = 2\cos\left(3x + \frac{\pi}{2}\right)$



131) $y = 3 \sin\left(\frac{1}{2}x + \frac{\pi}{4}\right)$



Use a calculator to find the value of the expression in radian measure rounded to 2 decimal places.

132) $\csc^{-1}\left(\frac{7}{2}\right)$

133) $\sec^{-1}\left(-\frac{8}{5}\right)$

134) $\cot^{-1}\left(-\frac{2}{3}\right)$

Using a calculator, approximate the value of the expression. Round answer to three decimal places.

135) $\sec^{-1}\left(-\frac{7}{3}\right)$

Simplify the expression as far as possible.

136) $\frac{\cos \theta}{1 + \sin \theta} + \tan \theta$

137) $(1 + \cot \theta)(1 - \cot \theta) - \csc^2 \theta$

Solve the equation for solutions in the interval $0 \le \theta < 2\pi$.

138) $\sin 4\theta = \frac{\sqrt{3}}{2}$

139) $\sqrt{2} \cos 2\theta = 1$

140) $5 \csc \theta - 2 = 3$

Use a calculator to solve the equation on the interval $0 \le \theta < 2\pi$. Round the answer to two decimal places.

141) $\tan \theta = 3.7$

SUMMER SOLUTIONS AP CALCULUS

1. The bit about a^2 is a red herring. We just need to know "a" is an integer. Multiplying something by 2 guarantees an even number. Adding one to this even number guarantees an odd. Ans:C

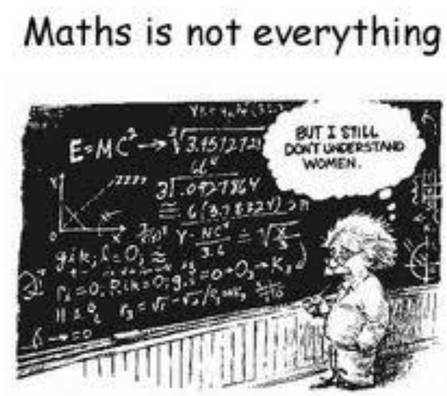
2. In calculus we often have to custom build something so substitute fit the format of a formula.

$$\frac{a}{b+1}=2$$

$$b+1=\frac{a}{2} \qquad \text{"move the numerator trick"} \qquad \frac{A}{B}=C \Leftrightarrow B=\frac{A}{C}$$

$$b=\frac{a}{2}-1 \qquad \text{now "build" what we need. } (2b) \text{ multiply by } 2$$ or

$$2b=a-2$$



3. I hope you didn't try to do this: Eqn 1 $5^2+(x+4)^2=13^2$

I REALLY REALLY hope you didn't then do this $5^2+x^2+4^2=13^2$ VERY BAD!!! (freshman binomial theorem.) If you started with Eqn 1, you have to square the binomial and get $5^2+x^2+8x+16=13^2$,which is inconvenient to solve because of the linear term.

I chose this problem because it is an excellent opportunity to use SUBSTITUTION to make the problem easier. Let M = x+4. Then its easy to solve for M. M= 12 so x+4 = 12 so x=8.

4.note that a side of the square is 2 radii. The square has side length 6 equal to the diameter or 2r. Perimeter = 4(side length) = 24.

$$A=\pi\cdot r^2$$

$$9\pi=\pi\cdot r^2$$

$$9=r^2 \Rightarrow r=\pm 3 \text{ only positive answer relevant}$$

5. D is the only one that follows.

Logic: another thing that penguins aren't very good at.

6. Note that similar figures imply that lengths are proportional. In common speech it means that you can multiply each length of the small by some number and get the length of the corresponding part of the big. We call this "multiply number" a constant of proportionality. Thus there exists a proportionality constant "k" such that…

$$b_2 = k \cdot b_1 \quad \& \, h_2 = k \cdot h_2$$

$$A_1 = \frac{1}{2} b_1 h_1 \qquad \& \qquad A_2 = \frac{1}{2} b_2 h_2$$

$$16 = \frac{1}{2} b_1 h_1 \qquad \& \qquad 36 = \frac{1}{2} k b_1 k b_2 \quad \textit{substitute: replace } b_2 \; \& \; h_2 \textit{ with what they equal.}$$

$$36 = \frac{1}{2} b_1 b_2 \cdot k^2$$

$$36 = 16 \cdot k^2$$

$$\frac{36}{16} = k^2 \Rightarrow k = \pm\frac{6}{4} = \pm\frac{3}{2}$$

Now let's say that the sides of the first triangle are A, B, and C. Then… Distribute backards!

$$Perimeter_1(P_1) = A + B + C \qquad and \quad Perimeter_2(P_2) = \frac{3}{2}A + \frac{3}{2}B + \frac{3}{2}C = \frac{3}{2}(A + B + C) = \frac{3}{2}P_1$$

$$so \quad P_2 = \frac{3}{2}P_1 \quad \text{divide both sides by } P_1 \text{ and flip both sides to get: } \frac{P_1}{P_2} = \frac{2}{3}$$

7.

$$f(x) = \frac{1}{2}x - c$$

$$f(8) = \frac{1}{2}(8) - c = 6$$

$$4 - c = 6$$

$$10 = c$$

8.

$$\left(3^{\frac{n}{2}}\right)^{-8} = 3$$

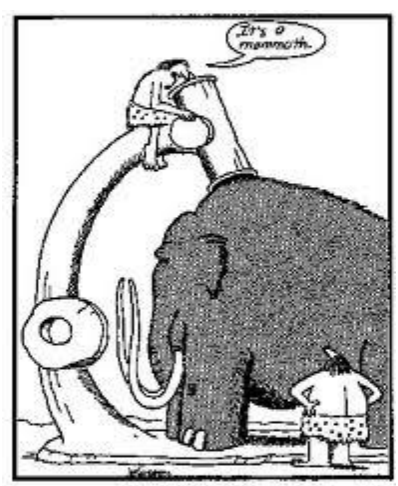
$$3^{-4n} = 3^1$$

$$-4n = 1$$

$$n = 1/4$$

(power to a power) = multiply.

This can only happen if the powers of three equal. Remember rules of exponents.



Early microscope

9. C=πd so the diameter of the big circle is ten and the diameter of the small circle is 6. Thus the radii are 5 & 3 respectively. “x” is the difference between the two 5-3=2

10. We are looking for a point that is 5 units away from (-1,3). They could have made this tough if we had to do the distance formula. Fortunately (-1,-2) works because the x number(-1)doesn’t change. We can find the distance by subtracting the y numbers 3-(-2) = 5, the desired radius.

11. The perimeter is 8+8+6+half the circumference. Remember

Circumference = πd; the diameter the same as the base of the rectangle (6). The circumference is 6π. The perimeter of the figure is 8+8+6+3π=22+3π

12.VERY important to be comfortable with the pattern of squaring a binomial:

$(X+Y)^2 = left^2 + twice\ product + right^2 = X^2 + 2XY + Y^2$ One has to notice that the left side has been squared so it will equal the square of the right side 1.2x1.2=1.44

13. Just substitute. Note that ½ has taken the place of a and 1 has taken the place of b so… $\dfrac{a+b}{a-b} = \dfrac{\frac{1}{2}+1}{\frac{1}{2}-1} = \dfrac{\frac{1}{2}+\frac{2}{2}}{\frac{1}{2}-\frac{2}{2}} = \dfrac{\frac{3}{2}}{\frac{-1}{2}} = \dfrac{3}{2}\cdot -\dfrac{2}{1} = -3$ Keep Flip Change

14. Perpendicular implies opposite and reciprocal slope. The given line has slope 2, so we need a slope of - ½ . I chose this problem to highlight the fact that

-1(x-y)= y-x: We can “flip” a difference by multiplying by negative one.

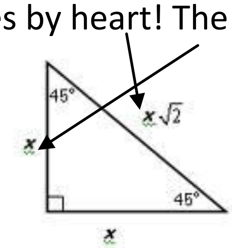
$$slope = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-7}{5-a} = \frac{-1}{2}$$

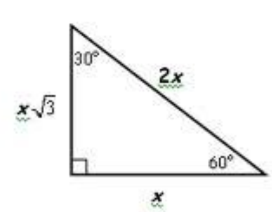
$$\begin{aligned} cross\,multiply \quad 2(4-7) &= -1(5-a) \\ -6 &= a-5 \\ -1 &= a \end{aligned}$$

15. NEEEED to know these special triangles by heart! The ratio of AB to BC is …

$\dfrac{x\sqrt{2}}{x} = \dfrac{\sqrt{2}}{1} = \sqrt{2} = \sec 45 = \csc 45$

(cancel the x’s) note:secx=hyp/adj; cscx=hyp/opp





16. They babied you on this giving a number (40) for the diameter. In fact the answer to this would be the same regardless of the diameter. Thus I will work it for the general case of diameter = d.

If the diameter of circle x is d, the radius of circle x is ½ d.

The radius of circle y is half of half of d or…½ (½ (d))= ¼ d. So…

$$radius\ \ of\ \ circle\ \ x\ \ is\ \ R_x = \frac{d}{2}\ \ \&\ \ radius\ of\ \ circle\ \ y\ \ is\ \ R_y = \frac{d}{4}$$

$$\left(you\ \ DO\ \ realize\ that\ \frac{1}{2}d = \frac{d}{2}\ \ \&\ \ \frac{1}{4}d = \frac{d}{4}.\ \ Right???\right)$$

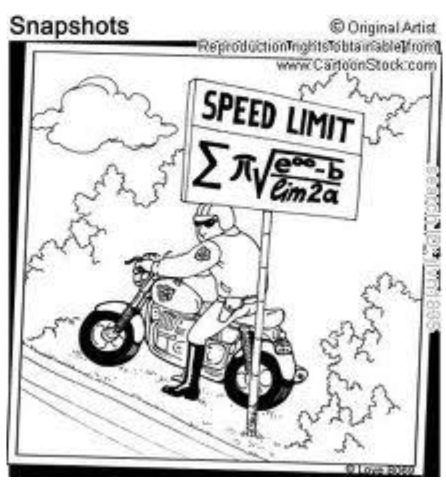
$$Area_{circle\ y} = \pi \cdot R_y^{\ 2} = \pi\left(\frac{d}{4}\right)^2 = \frac{\pi d^2}{16}\ \ square\ \ the\ \ top\ and\ \ bottom$$

$$Area_{circle\ x} = \pi \cdot R_x^{\ 2} = \pi\left(\frac{d}{2}\right)^2 = \frac{\pi d^2}{4}\ \ square\ \ the\ \ top\ and\ \ bottom$$

$$Ratio = \frac{Area_{circle\ y}}{Area_{circle\ x}} = \frac{\frac{\pi d^2}{16}}{\frac{\pi d^2}{4}} = \frac{\pi d^2}{16} \cdot \frac{4}{\pi d^2}\ \ now\ we\ cancel\ both\ \pi\ \&\ d^2$$

$$THIS\ \ IS\ \ WHY\ \ THE\ \ DIAMETER\ \ DOESN'T\ \ MATTER: D\ CANCELS$$

$$\frac{\cancel{\pi d^2}}{16} \cdot \frac{4}{\cancel{\pi d^2}} = \frac{4}{16} = \frac{1}{4}$$



17 This is a rate problem. We will spend a bunch of time working on a category of problems called "related rates problems" . You will see that calculus provides some powerful tools for problems involving rates. As a freshman, you learned d=rt (distance = rate x time). Now we generalize:

total whatever =(rate of whatever) x (time doing whatever)

or

total fences painted = (fences per minute) x (number of minutes painting)

The rate of painting when all three work together is the sum of the individual rates:

Yes this is seriously the answer. They will actually start doing stuff like this to you. Welcome to "big kid" math.

$$Rate\ \ of\ \ Ramon = R_R = \frac{1\,fence}{120\min}$$

$$Rate\ \ of\ \ Jenny = R_J = \frac{1\,fence}{105\min} \quad (fifteen\min\ \ faster)$$

$$Rate\ \ of\ \ Dennis = R_D = \frac{1\,fence}{60\min}\ (half\ \ Ramon's\ time)$$

$$Total\ rate\ when\ they\ work\ together:$$

$$\frac{1\,fence}{120\min}+\frac{1\,fence}{105\min}+\frac{1\,fence}{60\min}=\left(\frac{1}{120}+\frac{1}{105}+\frac{1}{60}\right)\frac{fence}{\min}=\frac{29\,fences}{840\min}$$

$$so....$$

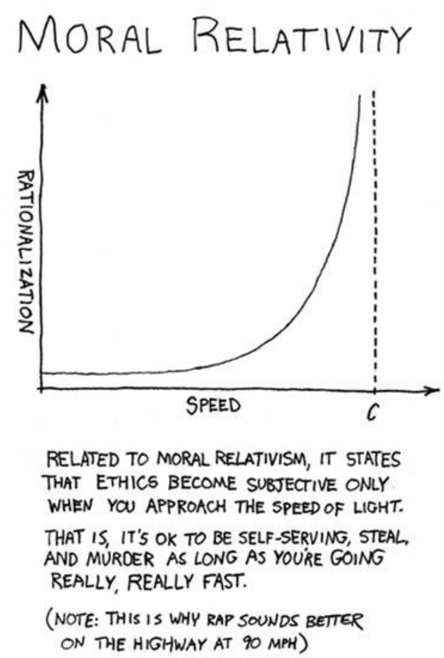
$$Total\ \ fences = rate\cdot time$$

$$1\ \ fence = \frac{29\,fences}{840\min}\cdot x\min \quad multiply both sides by reciprocal$$

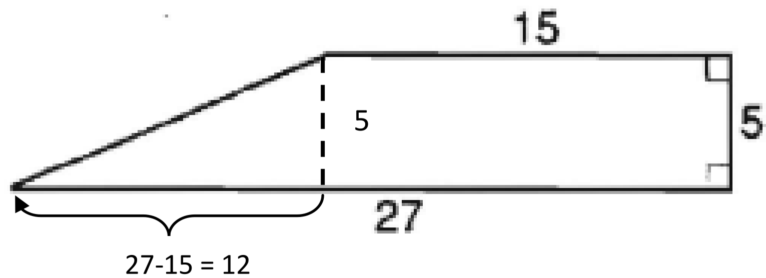
$$1\,\cancel{fence}\cdot\frac{840\min}{29\,\cancel{fences}}=\frac{\cancel{29\,fences}}{\cancel{840\min}}\cdot x\min\cdot\frac{\cancel{840\min}}{\cancel{29\,fences}} \quad cancel$$

$$\frac{840}{29}=x=28.96551724$$

$$x\ is\ about\ \ 29\ \min.$$



18. I chose this problem to illustrate that the opposite sides of a rectangle equal each other…Then it is a simple Pythagorean theorem problem (perimeter = sum of sides).



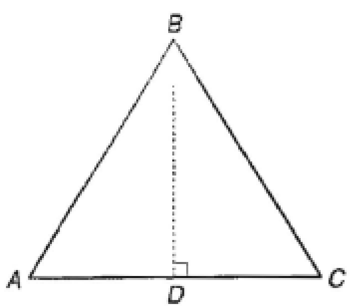
If you don't already have the 5, 12, 13 triangle memorized, its time now.

$$5^2 + 12^2 = x^2$$

Otherwise $169 = x^2$ Total perimeter 15+5+27+13=60

$$\pm 13 = x \text{ only positive root relevant}$$

19. We need to be 100% comfortable with slope.



if the slope of AB is 2/5 and the slope of BC is -2/5, the triangle is isosceles. i.e.

BD bisects AC

or equivilantly: AD = ½ AC (common sence or any of several proofs show this). Slope is a ratio: rise/run.

For AB $\frac{rise}{run} = \frac{BD}{AD} = \frac{2}{5}$

Well, if AC is 20 then AD is 10 (see above isosceles blah blah blah) so…

$$\frac{BD}{AD} = \frac{2}{5}$$
$$\frac{x}{10} = \frac{2}{5}$$
$$5x = 20$$
$$x = 4$$

20. Time to learn this if you don't know it already: $\frac{A}{B}\pm\frac{C}{D}=\frac{AD\pm BC}{BD}$. Don't freak out over the "plus or minus" ($\pm$). It just means that if the problem is addition, use addition, if subtraction use subtraction. This problem is addition…

$$\frac{A}{B}\pm\frac{C}{D}=\frac{AD\pm BC}{BD} \qquad \frac{1}{\sqrt{4x}}+\frac{\sqrt{x}}{2}=\frac{1\cdot 2+\sqrt{4x}\sqrt{x}}{\sqrt{4x}\cdot 2}$$

If you don't know that $\sqrt{whatever}\cdot\sqrt{whatever}=whatever$, *in this case* $\sqrt{x}\cdot\sqrt{x}=x$, Now is the time.

$$\frac{1}{\sqrt{4x}}+\frac{\sqrt{x}}{2}=\frac{1\cdot 2+\sqrt{4x}\sqrt{x}}{\sqrt{4x}\cdot 2}$$

$$=\frac{2+2x}{2\sqrt{x}\cdot 2}=\frac{\cancel{2}(1+x)}{\cancel{4}\sqrt{x}}$$ Simplify 2/4,

$$\frac{(1+x)}{2\sqrt{x}}\cdot\frac{\sqrt{x}}{\sqrt{x}}$$ Rationalize denominator.

$$\frac{\sqrt{x}+x\sqrt{x}}{2x}$$ Distribute the square root of x in the numerator.

21. Another rates problem: total = rate x time

What is his rate? 20x40 foot yard in 25 min is… $\frac{20ft\cdot 40ft}{25\min}=\frac{800}{25}\frac{ft^2}{\min}=32\frac{ft^2}{\min}$

Now the total is a 40x30 foot yard or 1200 square feet

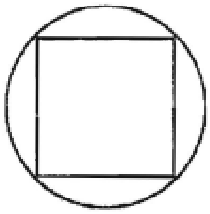
total = rate x time

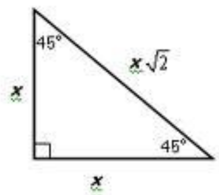
1200= 32 t

1200/32 = t

37.5 = t NOTICE THIS IS NOT THE ANSWER. It asks how many ***<u>MORE</u>*** minutes…
37.5(new)- 25(original) = 12.5

22. Note that the **diagonal of the square is the diameter of the circle** and that the diagonal of a square forms one of our essential 45, 45, 90 triangles. In this case… x = 10





Area of the square is 100.

Area of circle = $\pi \cdot r^2$. The diameter of the circle is $10\sqrt{2}$.

The radius is half the diameter or $5\sqrt{2}$ so

$$Area_{circle} = \pi \cdot r^2 = \pi \cdot \left(5\sqrt{2}\right)^2 = \pi \cdot 5^2 \sqrt{2}^2 = \pi 25 \cdot 2 = 50\pi$$

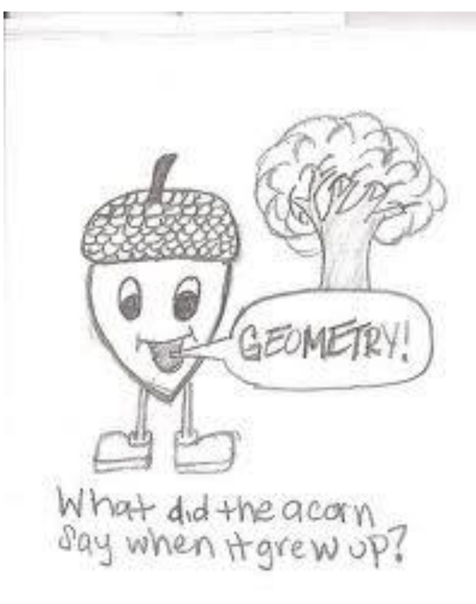
$$ratio \frac{areasquare}{areacircle} = \frac{100}{50\pi} = \frac{2}{\pi}$$

23 rules of logs Log(a/b) = Log a – log b. Rember log 10 = 1 & log 1 = 0

$$\log\frac{1}{10^a} = \log 1 - \log 10^a = 0 - \log 10^a$$

$$= -a\log 10 \quad remember\ that\ property \log x^A = A\log x$$

$$= -a \cdot 1 \quad = \quad -a$$



24. Synonyms for roots : zeros, x intercepts, roots solutions.

We solve these using the zero product principle. If two or more factors multiply to be zero, then at least one of them must be zero. We will turn the polynomial sum into a product. The verb “to factor” means to express as a product.

Group the first and last 2 terms.

They have common factors 4x squared and 9 respectively; factor these out.

Now the two terms have (x+3) in common. Factor that out.

Set each factor equal to zero. Remember i= square root of negative 1. ***Notice how I always use “plus or minus” when I take a square root of an equation, even when it is a geometric situation where the negative solution obviously doesn’t apply. It’s a good habit to maintain so you don’t leave out solutions in problems like this.***

$$\underline{4x^3+12x^2}+\underline{9x+27}=0$$

$$4x^2(x+3)+9(x+3)=0$$

$$(x+3)(4x^2+9)=0$$

$$x+3=0 \quad or \quad 4x^2+9=0$$

$$x=-3 \qquad -9 \quad -9$$

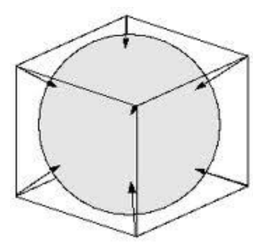
$$4x^2=-9$$

$$x^2=\frac{-9}{2}$$

$$x=\pm\sqrt{-\frac{9}{2}}=\pm\sqrt{-1}\sqrt{\frac{9}{2}}=\pm i\frac{3}{2}$$

$$x=\left\{-3,\frac{3}{2}i,-\frac{3}{2}i\right\}$$

25. The probability is the volume of the sphere divided by the volume of the cube. Like number 22, the size of them doesn’t matter.I will set the edge length of the cube equal to 2C. Why not C? Because I will soon be dealing with the volume of the sphere. The radius of the sphere is half of the edge length, so if I used C for the edge length, I would end up working with C/2. I try to avoid fractions when possible. If the edge is 2C, the radius of the sphere will be C.



Remember Keep flip change

$$\frac{volume\ sphere}{volume\ cube}=\frac{\frac{4}{3}\pi\cdot r^3}{edge^3}=\frac{\frac{4}{3}\pi\cdot C^3}{(2C)^3}=\frac{\frac{4}{3}\pi\cdot C^3}{8C^3}=\frac{\cancel{4\pi C^3}}{3}\cdot\frac{1}{\cancel{8C^3}}=\frac{1\pi}{3}\cdot\frac{1}{2}=\frac{\pi}{6}$$

26. These kind are a pain when you can't avoid two radicals on the same side. This makes you have to square both sides twice.

$$(X+Y)^2 = left^2 + twice\ product + right^2 = X^2 + 2XY + Y^2$$

$$\sqrt{ax+1} - \sqrt{ax-1} = \sqrt{ax}$$

$$\left(\sqrt{ax+1} - \sqrt{ax-1}\right)^2 = \left(\sqrt{ax}\right)^2$$

Squared cancels square root

$$\left(\sqrt{ax+1}\right)^2 - 2\sqrt{ax+1}\sqrt{ax-1} + \left(\sqrt{ax-1}\right)^2 = ax$$

$$ax + 1 - 2\sqrt{ax+1}\sqrt{ax-1} + ax - 1 = ax$$

Notice the +1 ans-1 cancel.

$$-ax \qquad -ax \qquad -2ax$$

Subtract off the two ax's

$$-2\sqrt{ax+1}\sqrt{ax-1} = -ax$$

$$\left(-2\sqrt{ax+1}\sqrt{ax-1}\right)^2 = (-ax)^2$$

$$4\cdot(ax+1)(ax-1) = a^2x^2$$

$$4(a^2x^2 - 1) = a^2x^2$$

Remember the difference of squares

$$4a^2x^2 - 4 = a^2x^2$$

$$-a^2x^2 + 4 \qquad -a^2x^2 + 4$$

$$3a^2x^2 = 4$$

$$a^2x^2 = \frac{4}{3}$$

$$\sqrt{a^2x^2} = \pm\sqrt{\frac{4}{3}}$$

Always use plus or minus.

$$ax = \pm\frac{2}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \pm\frac{2\sqrt{3}}{3}$$

$$\frac{1}{a}ax = \pm\frac{2\sqrt{3}}{3}\cdot\frac{1}{a}$$

$$x = \pm\frac{2\sqrt{3}}{3a}$$

Squaring an equation can make a falsehood true. For example: -3 = 3 if you square both sides it becomes true 9 = 9.

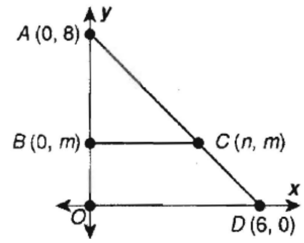
$$test\ -\frac{2\sqrt{3}}{3a}\ by\ substituting\ for\ x\ in\ top\ line\ right\ side$$

$$\sqrt{ax} \rightarrow \sqrt{\cancel{a}\left(-\frac{2\sqrt{3}}{3\cancel{a}}\right)} = \sqrt{-\frac{2\sqrt{3}}{3}} = imaginary\ answer.$$

Always check solutions when you square both sides.

$$so\ \ only\ \ answer\ is \qquad x = \frac{2\sqrt{3}}{3a}$$

27. ***The main point here the "x" coordinate controls the "over" and the "y" coordinate controls the "up".*** Recall the formulae for the areas of triangles and trapezoids.



$$Triangle\,Area = \frac{1}{2}bh \quad \& \quad Trapezoid\,area = \frac{base_1 + base_2}{2} \cdot h$$

The base of triangle ABC is n since B is "over" zero and C is over n (n-0=n). They are both "up" the same (m). The height of triangle ABC is 8-m since A is "up" 8 and B is "up" m. They are both over the same (zero).

$$Area\ triangle ABC\ \frac{1}{2}bh = \frac{1}{2}n(8-m)$$

By similar reasoning, the top base of the trapezoid is n (it is the same segment as the base of triangle ABC) The bottom base of the trapezoid is 6. The height of the trapezoid is m since B and C are both "up" m.

$$area\ trapezoid OBCD = \frac{B_1 + B_2}{2} \cdot h = \frac{(n+6)}{2} \cdot m$$

Triangle AOD has area 24 = ½ 6(8). Both triangle ABC and trapezoid OBCD have area equal to one half triangle AOD (12). Set up two equations. Solve both for n.

$$12 = \frac{1}{2}n(8-m) \qquad \& \qquad 12 = \frac{n+6}{2}m$$

$$24 = n(8-m) \qquad \& \qquad 24 = (n+6)m$$

$$\frac{24}{8-m} = n \qquad \& \qquad \frac{24}{m} = n+6$$

$$-6 \quad -6$$

If these two expressions both equal n, they must equal each other.

$$\frac{24}{8-m} = n \qquad \& \qquad \frac{24}{m} - 6 = n$$

$$\frac{24}{8-m} = \frac{24}{m} - 6$$

Excellent technique: multiply by the denominators m(8-m) to clear fractions.

$$\left(\frac{24}{8-m} = \frac{24}{m} - 6\right)(8-m)(m)$$

$$24m = 24(8-m) - 6m(8-m)$$

$$4m = 4(8-m) - m(8-m)$$

$$4m = 32 - 4m - 8m + m^2$$

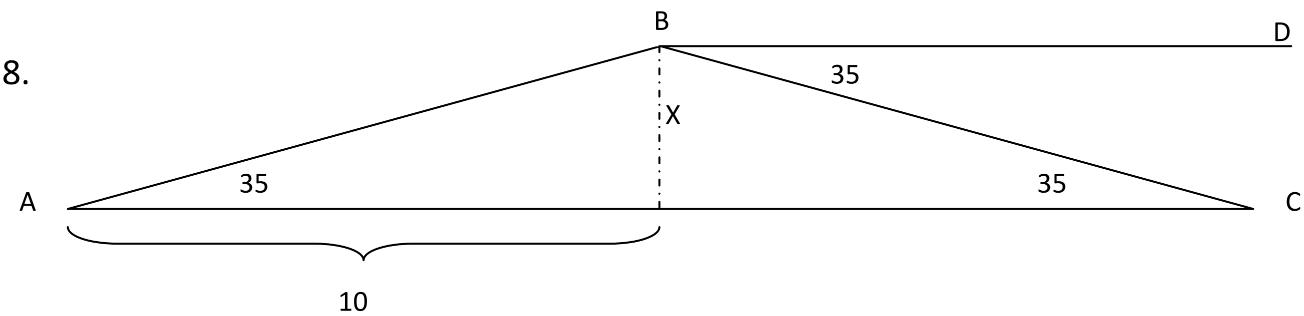
$$0 = m^2 - 16m + 32$$

$$m = \frac{-(-16) \pm \sqrt{(-16)^2 - 4(1)(32)}}{2(1)} = \frac{16 \pm \sqrt{128}}{2}$$

$$m = \frac{16 \pm \sqrt{64} \cdot \sqrt{2}}{2} = \frac{16 + 8\sqrt{2}}{2} = 8 \pm 4\sqrt{2}$$

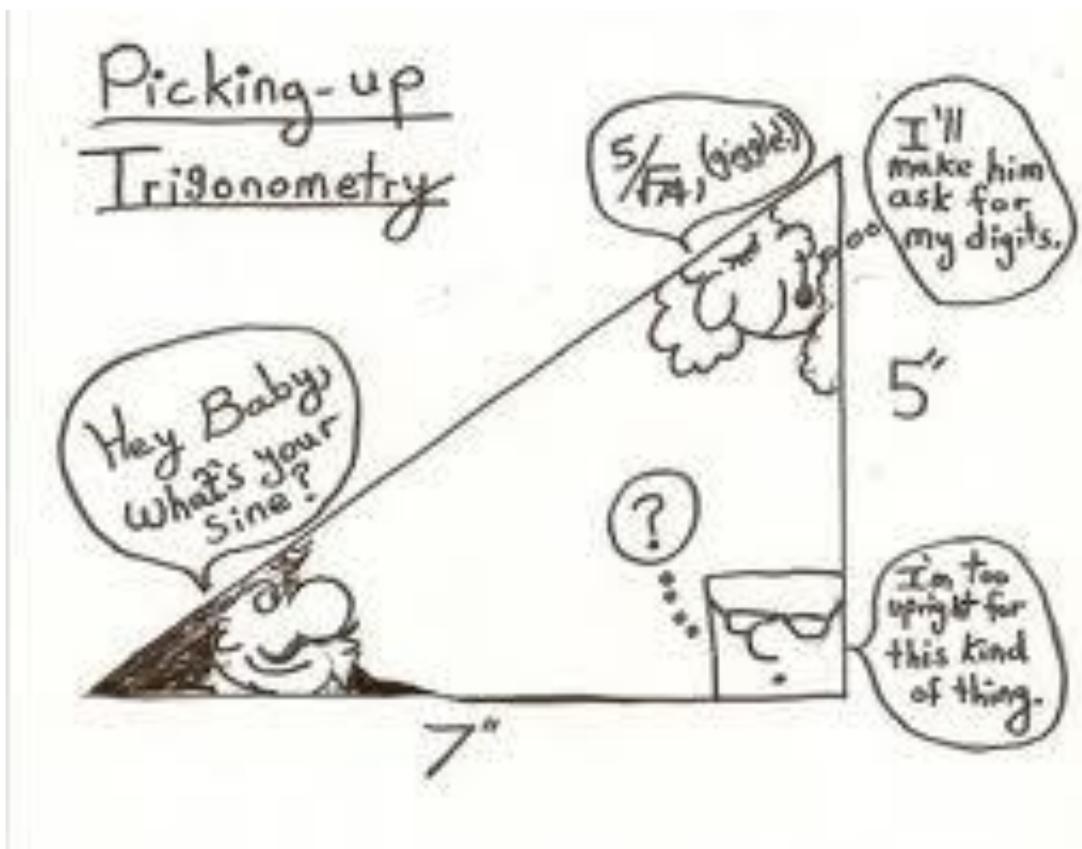
We must look at solutions to test for reasonableness. $8 + 4\sqrt{2}$ is about 13.7, which by looking at the diagram is not applicable. M must be $8 - 4\sqrt{2}$ (About 2.34)

28.



Observe that the triangle is isosceles since angle of depression <DBC is alternate interior with <BCA, thus the altitude at B bisects the base. On trig questions, the first question to ask oneself is “What two sides are involved?” (opposite and adjacent). Then “Which trig function is that: (tan). Remember SOHCAHTOA?

$\tan\theta = \frac{opp}{adj} \Rightarrow \tan(35) = \frac{x}{10} \Rightarrow x = 10\tan 35 \approx 7.00\,miles$ Remember to be in degrees in your calculator(under “mode”) 90% of the time in this class we will be in radians. Calculus will only work with radians.



29. To understand what's going on, consider a simpler example.

$$A+\frac{3}{14}=\frac{26}{70}$$
$$-\frac{3}{14}\quad -\frac{3}{14}$$
$$A=\frac{26}{70}-\frac{3}{14}$$
$$A=\frac{26}{70}-\frac{3}{14}\cdot\frac{5}{5}$$
$$A=\frac{26}{70}-\frac{15}{70}=\frac{9}{70}$$

$$14\overline{)70}\quad 5$$

To find out what is "missing" from the 14 to be a common denominator with 70, we can divide 70 divided by 14. We know this will divide evenly since 70 was apparently the common denominator with 14 and whatever the denominator of A is.

$$\frac{2x-3}{3x^2+16x+5}+A=\frac{3x^2+3x+18}{3x^3+13x^2-11x-5}$$

$$-\frac{2x-3}{3x^2+16x+5}\qquad -\frac{2x-3}{3x^2+16x+5}$$

$$A=\frac{3x^2+3x+18}{3x^3+13x^2-11x-5}-\frac{2x-3}{3x^2+16x+5}$$

$$\begin{array}{r} 1x \quad - \quad 1 \\ 3x^2+16x+5\overline{)3x^3+13x^2-11x-5} \\ -\left(3x^3+16x^2+5x\right) \\ -3x^2-16x-5 \\ -\left(-3x^2-16x-5\right) \\ 0 \end{array}$$

$$A=\frac{3x^2+3x+18}{3x^3+13x^2-11x-5}-\frac{2x-3}{3x^2+16x+5}\,\frac{(x-1)}{(x-1)}$$

$$A=\frac{3x^2+3x+18}{3x^3+13x^2-11x-5}-\frac{2x^2-2x-3x+3}{3x^3+13x^2-11x-5}$$

$$A=\frac{3x^2+3x+18-\left(2x^2-2x-3x+3\right)}{3x^3+13x^2-11x-5}$$

$$A=\frac{3x^2+3x+18-2x^2+2x+3x-3}{3x^3+13x^2-11x-5}=\frac{x^2+8x+15}{3x^3+13x^2-11x-5}$$

But now we notice that the numerator factors. $x^2+8x+15=(x+3)(x+5)$

And from the earlier division, the denominator can be factored:

The quadratic factor in the denominator can be factored with the "rainbow method"* see below*

$$\frac{x^2+8x+15}{3x^3+13x^2-11x-5}=\frac{(x+3)(x+5)}{(1x-1)(3x^2+16x+5)}=\frac{(x+3)(x+5)}{(1x-1)(3x+1)(x+5)}=\frac{(x+3)}{(1x-1)(3x+1)}$$

REVIEW of the RAINBOW METHOD!

$$6x^2 + 7x - 3$$

Multiply the coefficients of the x^2 and the constant terms.

$$6 \cdot -3 = -18$$

Find two numbers whose product is this number and whose sum is the coefficient of the x term.

$$9 \cdot -2 = -18$$
$$9 + -2 = 7$$

$$6x^2 + 9x - 2x - 3$$

Replace the x term with two x terms whose coefficients are the two numbers you found in the first step.

$$6x^2 + 9x \mid -2x - 3$$

Divide your expression between the two x terms.

$$3x(2x + 3) - 1(2x + 3)$$

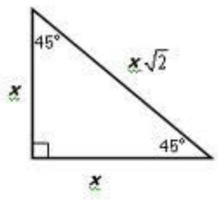
Factor a GCF out of each "half" of the expression. (Both resulting binomials should be the same. WOW!)

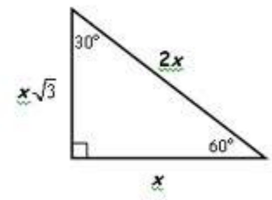
$$(2x + 3)(3x - 1)$$

Factor this common binomial out of each "half".

30. Arc AB is half the circumference. The whole circumference must have been 32π. Thus, C=πd ; 32π=πd implies that d=32. So segment AB is the diameter= 32

Remember our old friends: Set up a proportion. Remember to rationalize the denominator by multiplying by 1 in the form of root 2 over root 2. Remember that root two times root two is two





$$\frac{BC}{32}=\frac{x}{x\sqrt{2}}$$

$$BC\sqrt{2}=32$$

$$BC \quad =\frac{32}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=\frac{32\sqrt{2}}{2}=16\sqrt{2}$$

31.The domain will be all x numbers except those that make a denominator zero or a negative value in an even root. In this case there are no radicals, so we just have to check for a zero denominator. The numbers that make the denominator zero are not allowed.

$$2x^2-11-6=0$$

$$2x^2-17=0$$

$$2x^2=17$$

$$x^2=\frac{17}{2}$$

$$x=\pm\frac{\sqrt{17}}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=\pm\frac{\sqrt{34}}{2}$$

$$thus\ D=\left\{x: x\in R, x\neq\pm\frac{\sqrt{34}}{2}\right\}$$

32. A denominator cannot EVER make a fraction equal zero! So we just need to look at what makes the numerator zero.

$$x^2+3x+2=0$$

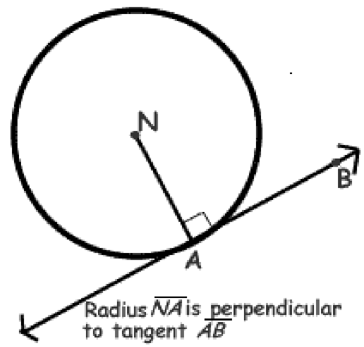
$$(x+2)(x+1)=1$$

$$x+2=0 \quad or \quad x+1=0$$

$$x=-2 \quad or \quad x=-1$$

The sequence is -1, -2, -3, -5, -8, -13, -21, ….. The fifth term is -8

33. Remember that a tangent is perpendicular to a radius. So we need a line perpendicular to the y=-3x+7 that passes through the origin. (y intercept will be zero) remember that the slope of a line perpendicular is the opposite reciprocal. The given line has slope -3, so we need a slope of 1/3 . Since the y intercept is zero, the equation of the line we seek is y= 1/3 x. We need to find the point of intersection between y= 1/3x and the given line y=-3x+7. Since both equations are solved for y, substitution is a convenient method: if we have two expressions both equal to y, they must equal each other. Set them equal.



$$\frac{1}{3}x = -3x + 7$$
$$x = -9x + 21$$
$$10x = 21$$
$$x = 2.1$$
$$y = -3x + 7$$
$$y = -3(2.1) + 7$$
$$y = -6.3 + 7 = 0.7$$

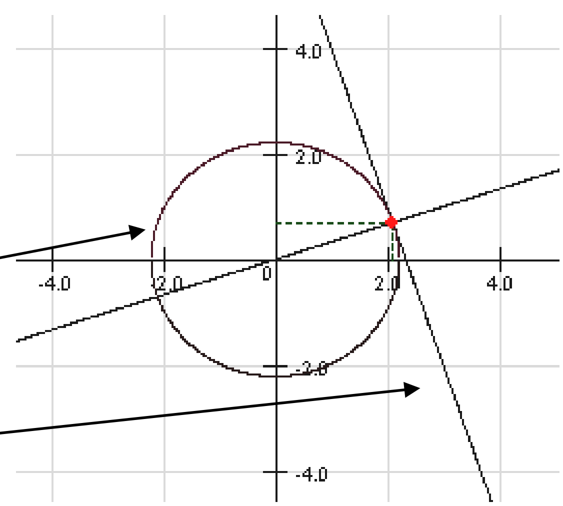
Solving is much more convenient if one multiplies by a number to clear the denominators; in this case three.

Once we know x, we substitute it in for x in one of the original equations to find y. The point of intersection is (2.1,0.7). To find the radius, use the distance formula to find the distance between the origin and the point of intersection.

$$D = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$
$$= \sqrt{(0.7-0)^2 + (2.1-0)^2}$$
$$= \sqrt{\left(\frac{7}{10}\right)^2 + \left(\frac{21}{10}\right)^2} = \sqrt{\frac{49}{100} + \frac{441}{100}} = \sqrt{\frac{490}{100}} = \frac{\sqrt{49}\sqrt{10}}{\sqrt{100}} = \frac{7\sqrt{10}}{10}$$

The equation for a circle with center (h,k) and radius "r" is . $(x-h)^2 + (y-k)^2 = r^2$ Our center is (0,0) and radius is $\frac{7\sqrt{10}}{10}$. So we have:

$$(x-0)^2 + (y-0)^2 = \left(\frac{7\sqrt{10}}{10}\right)^2 = x^2 + y^2 = \frac{49 \cdot 10}{100}$$
$$x^2 + y^2 = 4.9$$
$$y = -\frac{1}{3}x$$
$$y = -3x + 7$$



"I plead the fifth."

composition of functions is NOT commutative: (order matters)

Example. Let f(x)=2x+1 and g(x) = -4x

f(g(x)))=2(-4x)+1=-8x+1 "Put g's formula into f's formula"

g(f(x))=-4(2x+1)=-8x-4 "Put f's formula into g's formula"

NOT THE SAME!!!

Now it is true that the composition of a function and its inverse will always be f(x)=x : in other words, it gives back what you put in.

Example: let f(x) = 4x-2. Find the inverse function and compose the functions.

$$f(x) = 4x-2$$
$$y = 4x-2$$
$$x=4y-2$$
$$x+2=4y$$
$$\frac{x+2}{4}=y$$
$$f^{-1}(x)=\frac{x+2}{4}$$
$$f(f^{-1}(x)) = 4\left(\frac{x+2}{4}\right)-2 \quad \& \quad f^{-1}(f(x))=\frac{(4x-2)+2}{4}$$
$$f(f^{-1}(x)) = \cancel{4}\left(\frac{x+2}{\cancel{4}}\right)-2 \quad \& \quad f^{-1}(f(x))=\frac{4x\cancel{-2}\cancel{+2}}{4}$$
$$f(f^{-1}(x)) = x+\cancel{2}-\cancel{2} \quad \& \quad f^{-1}(f(x))=\frac{\cancel{4}x}{\cancel{4}}$$
$$f(f^{-1}(x)) = x \quad \& \quad f^{-1}(f(x))=x$$

Remember to find the inverse function.

1. replace f(x) with y

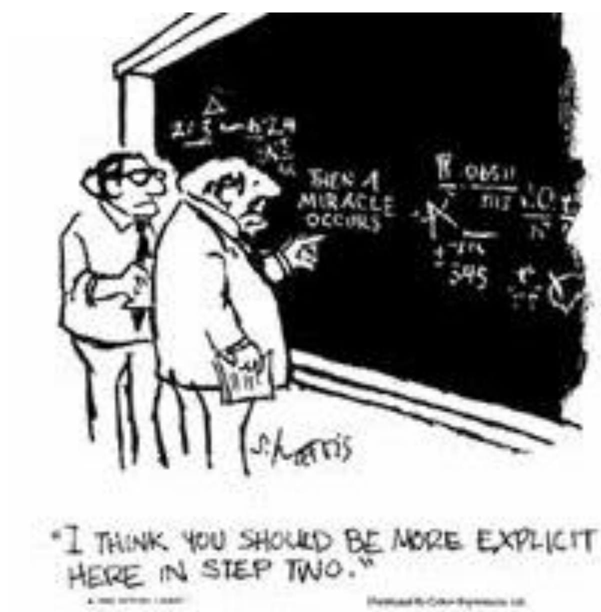
2.switch x and y

3. solve for y again

4. this is your inverse!

EXCEPTION to the rule!!! When composing a function with its inverse, it gives "x" in either order.

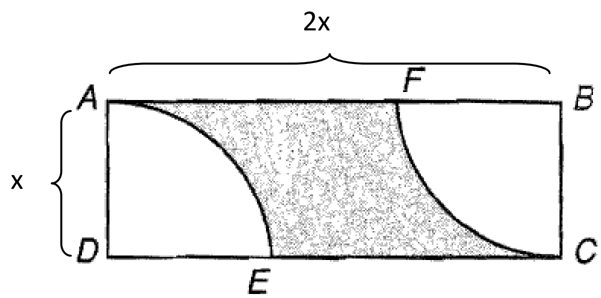
"A" is the right choice. If you do f(g) first you get h(x) …see the question. Then if you do h inverse to h you get back x as above.



36.Shaded area = full rectangle area – circle parts area

NOTE: we have 2 quarter circles for a total of a half circle; The radius of the circles is x, length = 2x, width = x

$$\begin{aligned}
&AREA \text{ rectangle} = lw \\
&Area\ circle \quad = \pi \cdot r^2 \\
&half\ circle \quad = \frac{\pi \cdot r^2}{2} \\
&\text{rectangle} - halfcircle \\
&2x \cdot x \quad - \frac{\pi \cdot x^2}{2} \\
&2x^2 \quad - \frac{\pi \cdot x^2}{2} \\
&\frac{2}{2} \cdot \frac{2x^2}{1} - \frac{\pi \cdot x^2}{2} \\
&\frac{4x^2}{2} - \frac{\pi \cdot x^2}{2} = \frac{4x^2 - \pi \cdot x^2}{2} = \left(\frac{4-\pi}{2}\right)x^2
\end{aligned}$$

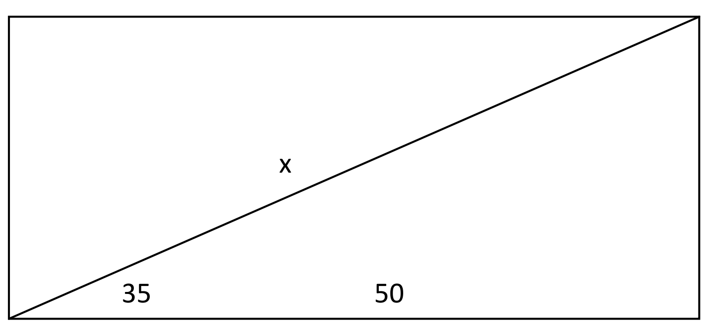


37. We are building what we need: how to turn y into y squared +6. Just do the same thing to both sides. Remember the pattern for squaring binomials

$$(X+Y)^2 = left^2 + twice\ product + right^2 = X^2 + 2XY + Y^2$$

$$\begin{aligned}
&3x - 2 = y \\
&(3x-2)^2 = y^2 \\
&9x^2 - 12x + 4 = y^2 \\
&9x^2 - 12x + 4 + 6 = y^2 + 6 \\
&9x^2 - 12x + 10 = y^2 + 6
\end{aligned}$$

38.



$$\cos(35) = \frac{50}{x} \quad \Rightarrow \quad \frac{50}{\cos(35)} = x \approx 61.0$$

39. IMPORTANT DEFINITION TO THE RIGHT!!!

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

Remember "less thAND" and "greatOR"

This one is less thAND

- Your algebra 1 teacher probably had you skip to here.
- It is safer in difficult math to express and follow the definition above you probably think I'm crazy now, but you will see.
- Remember to flip the inequality if you multiply or divide by a negative

$$|2x+1|+3<8$$

$$\begin{array}{ccc} \text{if } 2x+1 \geq 0, & & \text{if } 2x+1<0 \\ x \geq -\dfrac{1}{2} & & x < -\dfrac{1}{2} \\ 2x+1+3<8 & AND & -(2x+1)<8 \\ 2x+4<8 & & 2x+1>-8 \\ 2x<4 & & 2x>-9 \\ x<2 & & x>-\dfrac{9}{2} \end{array}$$

$$x \in \left(\frac{-9}{2}, 2\right)$$

- Using the interval notation shown, round brackets is < or >, rectangular brackets are "less than or equal to" or "greater than or equal to"

40. remember the squaring a binomial pattern

$$(X+Y)^2 = left^2 + twice\ product + right^2 = X^2 + 2XY + Y^2$$

This problem would be formidable if you weren't always ready to do the binomial pattern. IT IS VERY IMPORTANT!!!

$$4m^2+9n^2=1 \qquad (2m-3n)^2=13$$

$$4m^2-12mn+9n^2=13$$

$$4m^2+9n^2-12mn=13$$

$$1 \quad -12mn=13$$

$$-1 \qquad\qquad -1$$

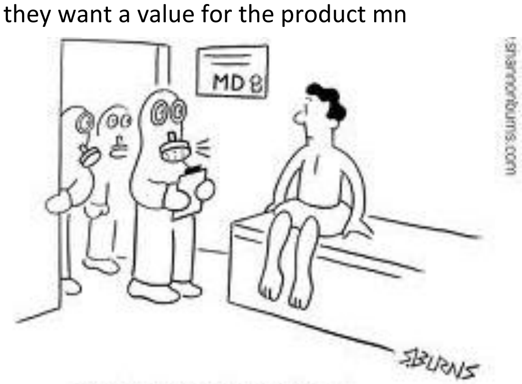
$$-12mn=12$$

$$mn=\frac{12}{-12}=-1$$

Learn to be flexible with substitution.

rearrange

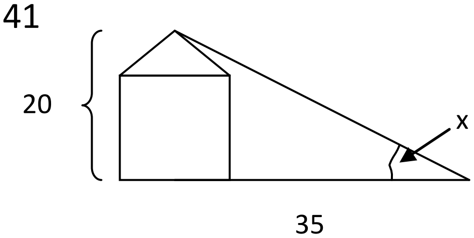
See how we substitute one for the whole glob $4m^2+9n^2$. See how we are not always solving for a single variable any more. Here they want a value for the product mn



"Here's what we know so far..."

40 B Piecewise function: just know that we are in the lower category. 3 is not >3. 3 is less than or equal to 3. Just substitute 3 into lower expression (3)+2=5

41



$$\tan x = \frac{20}{35}$$

$$\tan^{-1}(\tan x) = \tan^{-1}\left(\frac{20}{35}\right)$$

$$x \approx 30$$

On trig questions, the first question to ask oneself is “What two sides are involved?” (opposite and adjacent). Then “Which trig function is that: (tan). Remember SOHCAHTOA? Remember to be in degrees in your calculator(under “mode”). We use inverse trig functions “get rid of” trig functions (to find angles). They are above the regular trig functions. (2^{nd} tan)

42The important points are that a line has the same slope no matter where you measure it & that this line passes through the origin. (0,0)

Its slope is $\frac{y_2 - y_1}{x_x - x_1} = \frac{5-0}{4-0} = \frac{5}{4}$.

So just redo the slope formula with the points (y,ay) and (4,5):

$$\frac{y_2 - y_1}{x_x - x_1} = \frac{5-ay}{4-y} = \frac{5}{4}$$

$$4(5-ay) = 5(4-y)$$

$$20 - 4ay = 20 - 5y$$

$$-20 \qquad -20$$

$$-4ay = -5y$$

$$a = \frac{-5y}{-4y} = \frac{5}{4}$$

Divide by the -4 and y at the same time. Dontcha just love it when stuff cancels.

This problem is an example of direct variation: every time you divide a y coordinate on the line by its x coordinate you get the same answer (well O.K. not at (0,0) smarty pants!)

43. Increasing [0,3)

Decreasing(3,6] Notice at 3 it is neither increasing nor decreasing.

The maximum occurs at time 3.

44. we need to build a 5(x/y) yippee!

$$\begin{aligned} 5x-6 &= 3(y-2) \\ 5x-6 &= 3y-6 \\ +6 \quad & \quad +6 \\ 5x &= 3y \\ 5\frac{x}{y} &= 3 \end{aligned}$$

45.since it is a line (doesn't bend), we can study its limit points: g(-4), g(0), g(4)

g(-4)=-3(-4)+5=17

g(0)=-3(0)+5=5

g(4)=3(4)+6=18   range [5,18]

$$\sin(ax) = 0.3\cos(ax) \qquad REMEMBER\ \frac{\sin A}{\cos A} = \tan A$$

$$\frac{\sin(ax)}{\cos(ax)} - 0.3$$

46. $\tan(ax) = 0.3$

$$\tan^{-1}(\tan(ax)) = \tan^{-1}(0.3) \quad get\ in\ radians!\ \text{"mode"}$$

$$ax \approx 0.29$$

$$a \approx \frac{0.29}{x}$$